Bilateral Trade with Interdependent Values

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Bilateral trade

Setting: A seller with one item and a potential buyer. The seller values the item v_s and the buyer values it v_b, and these values are **private information**, drawn from **publicly** known distributions.

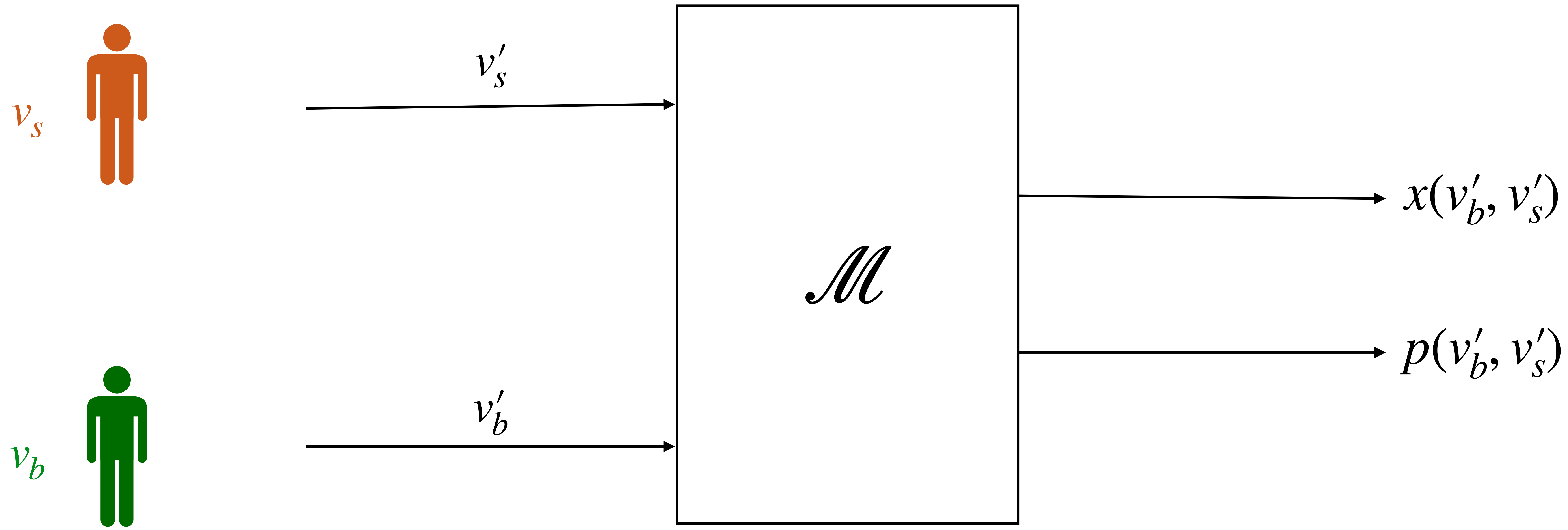
Designer goal: Decide if the trade should happen and under what payment scheme.

Natural objective: Trade whenever the buyer values the item more than the seller ($v_b > v_s$).

Mechanism Design Task

A (direct) **mechanism** $\mathcal{M}$ consists of two functions $\mathcal{M} = (x, p)$, takes **reported** values v'_b and v'_s as input and outputs:

1. The **probability** of trade $x(v'_b, v'_s)$.
2. The **price** that the buyer pays to the seller $p(v'_b, v'_s)$.



Utilities & Mechanism Constraints

Utilities under mechanism $\mathscr{M}$ with reports (v_b', v_s'):

Seller utility: $p(v_s', v_b') - v_s \cdot x(v_s', v_b')$

Buyer Utility: $v_b \cdot x(v_s', v_b') - p(v_s', v_b')$

Desired Mechanism Constraints:

1. **Individual Rationality** (IR) -----> Non-negative utility from participating in the mechanism.
2. **Incentive Compatibility** (IC) --> No incentive to misreport my information to the mechanism.
3. **Budget Balance** (BB) ------------> The designer does not subsidize the trade.

Objectives

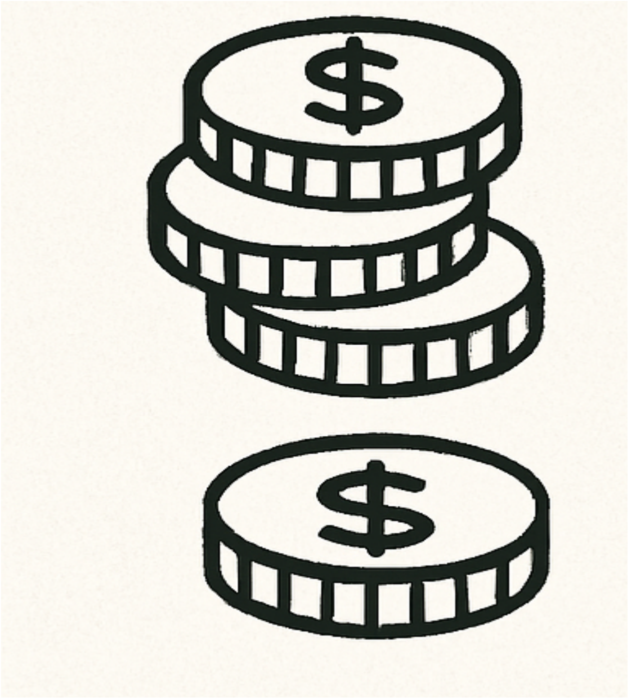
A mechanism's **performance** in bilateral trade is commonly measured in:

1. Social Welfare: the **value** (welfare) of the agent that is allocated the item.

   The benchmark to compare against is $SW = \mathbb{E}\left[max(v_b, v_s)\right]$.

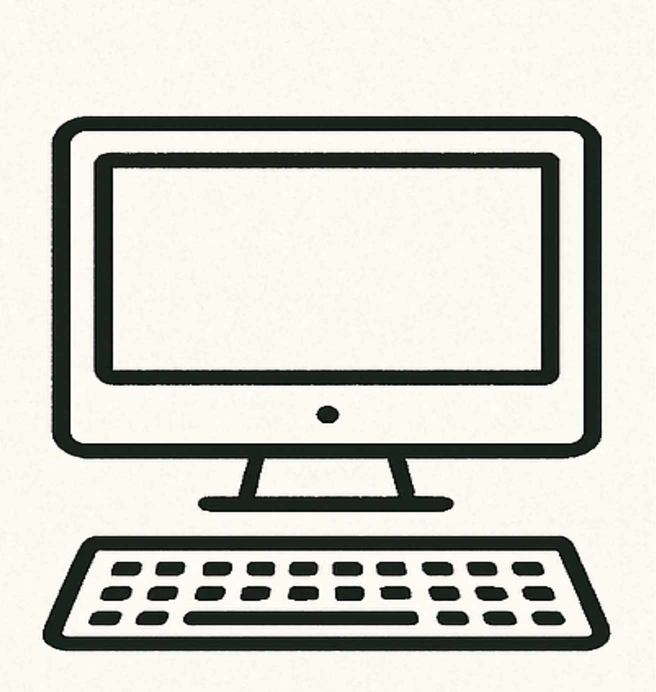
2. Gains from Trade: the **welfare increase** due to the trade (if it happens).

   The benchmark to compare against is $GFT = \mathbb{E}\left[max(v_b - v_s, 0)\right]$.



Economists: Optimality is unattainable

(Informal) **Theorem [Myerson-Satterthwaite '83]:** There exists no mechanism that simultaneously guarantees individual rationality, incentive compatibility, budget balance, and maximizes Social Welfare.



Computer Scientists: Approximation Thrives

Independent Values:

- Welfare:
  - Multiple works with posted price mechanisms [Blumrosen and Dobzinski, 2014, 2021, **Cai and Wu, 2023**, Colini-Baldeschi et al., 2016, **Kang et al., 2022**, **Liu et al., 2023**].
  - State of the Art (**blue**) is a **1.38** approximation, Lower bound (**red**) is **1.354**.
- Gains From Trade:
  - Again multiple works with posted price mechanisms [**Babaioff et al., 2021**, 2020, Blumrosen and Dobzinski, 2014, Brustle et al., 2017, Cai et al., 2021, Deng et al., 2022, **Fei, 2022**, McAfee, 2008]
  - State of the Art (**blue**) achieves a **3.15** approximation, Lower bound (**red**) is **1.358**.

Correlated Values - Welfare:

- Single work [Dobzinski and Shaulker, 2024], that proves that a posted price mechanism achieves a **tight** **1.582** approximation.

A realistic scenario

An art connoisseur is considering selling their marble sculpture to a civil engineer.

Trade?

Knows Signal s:
The "artistic" value of the sculpture.

Knows Signal b:
The "material" value of the sculpture.

What if they both care about the other agent's information?

Interdependent Values in Bilateral Trade

The seller has a **private** signal s and the buyer has a **private** signal b. The signals are drawn from **publicly** known distributions.

Their values for the item are **public functions** of the signals, that is the seller's value is $v_s(b, s)$ and the buyer's value is $v_b(b, s)$.

Motivating Interdependent Values

- This is an established model, formalized by **Milgrom & Weber** in 1982.
- The model can be used to describe both partial information and information asymmetry, it naturally **generalizes** the **independent** and **correlated** values model, while also capturing unaccounted for **real world** scenarios.
- **Examples:**

1. Resale values, Values with revision effects [Myerson '81].
2. Common values - Mineral Rights model [Wilson 1969], Wallet game [Klemperer 1998].

(Seller) Incentive Constraints

- The buyer reports their true signal b.
- The seller reports signal s' while their true signal is s.
- The seller utility is $U_S(s', s, b) = p(b, s') - v_s(s, b) \cdot x(b, s')$.

Bayesian Incentive Compatibility (BIC): A mechanism $\mathcal{M} = (x, p)$ satisfies bayesian incentive-compatibility for the seller if for every (s, s') in the seller domain:

$$\mathbb{E}_b \left[U_S(s, s, b) \right] \geq \mathbb{E}_b \left[U_S(s', s, b) \right]$$

Interim Individual Rationality (Interim IR): A mechanism $\mathcal{M} = (x, p)$ satisfies interim individual-rationality for the seller if for every s in the seller domain:

$$\mathbb{E}_b \left[U_S(s, s, b) \right] \geq 0$$

Posted Prices & Interdependent Values

- Under **independent/correlated** values posted price mechanisms are trivially DSIC and ex-post IR, and always result in trade if $v_s < p < v_b$.
- Under **interdependent** values these properties are **no longer guaranteed**.

- Suppose $b = 0.5$ and a posted price $p = 1.5$; The buyer has no dominant strategy!

$v_s = 2 \cdot b + s$

$s \sim U[0,1]$

Price $p = 1.5$

$v_b = 2 \cdot s + b$

$b \sim U[0,1]$

(α, β)-Information Structures

- (For ease of presentation) We restrict to **additively separable valuations**, with signals b, s drawn **independently** from $U[0,1]$:

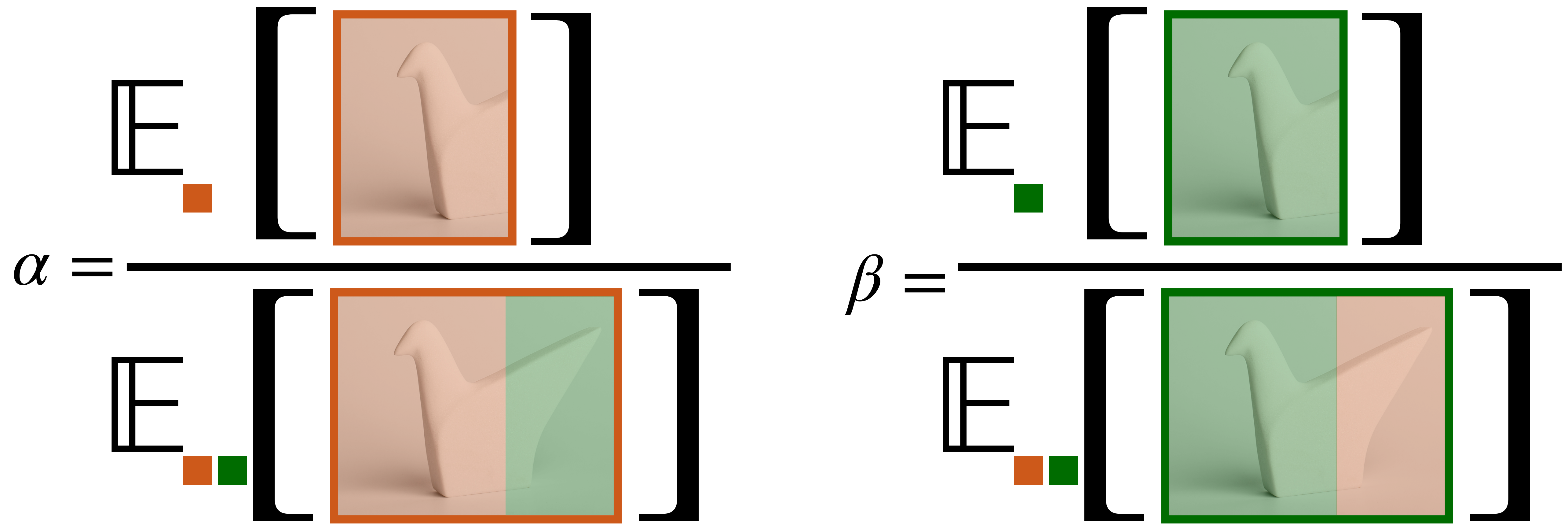
$$v_s(b, s) = f_s(b) + g_s(s), \qquad v_b(b, s) = f_b(b) + g_b(s),$$

  where, $f(\cdot), g(\cdot)$ are non-negative, increasing functions.

- We define parameters α (seller) and β (buyer) that quantify the **influence** that an agent's **private signal has on their own valuation.** We provide results for (α, β)-information structures.

Seller and Buyer Informedness

- **Defining (α,β) pictorially:**

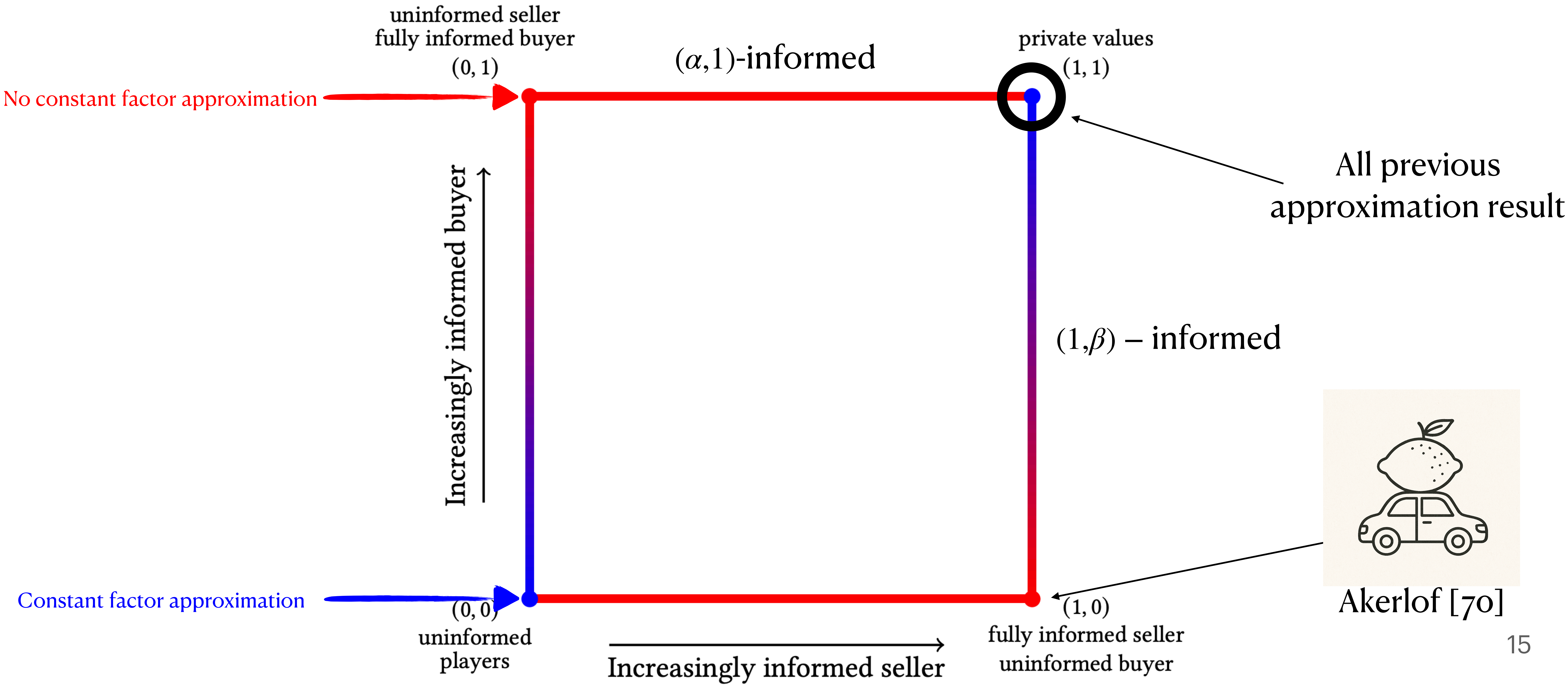


- **Uninformed** seller corresponds to $\alpha = 0$. **Fully informed** seller corresponds to $\alpha = 1$.

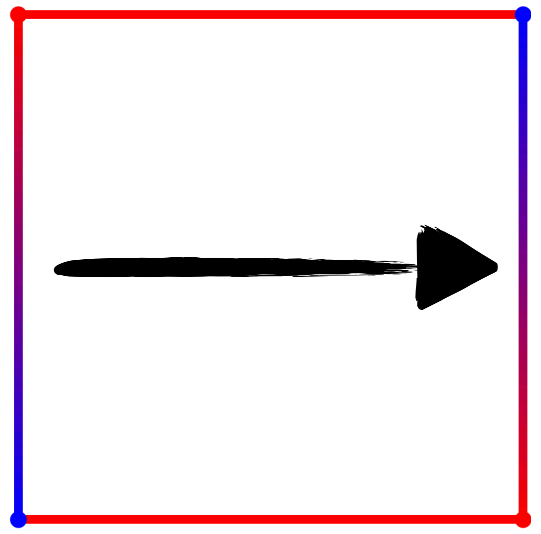
- Formally, we denote the seller α**-informed** and the buyer β**-informed** with:

$$\alpha = \frac{\mathbb{E}_s[v_s(0,s)]}{\mathbb{E}_{s,b}[v_s(b,s)]}, \qquad \beta = \frac{\mathbb{E}_b[v_b(b,0)]}{\mathbb{E}_{b,s}[v_b(b,s)]}.$$

Overview of results for (α,β) - information structures on the square

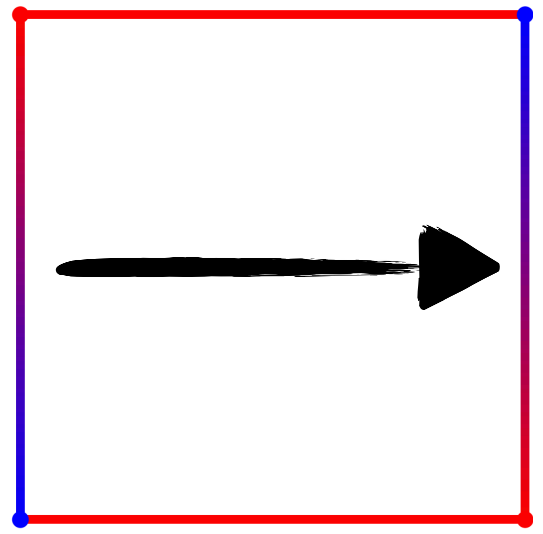


A Fully Informed Seller: $(1, \beta)$ Edge

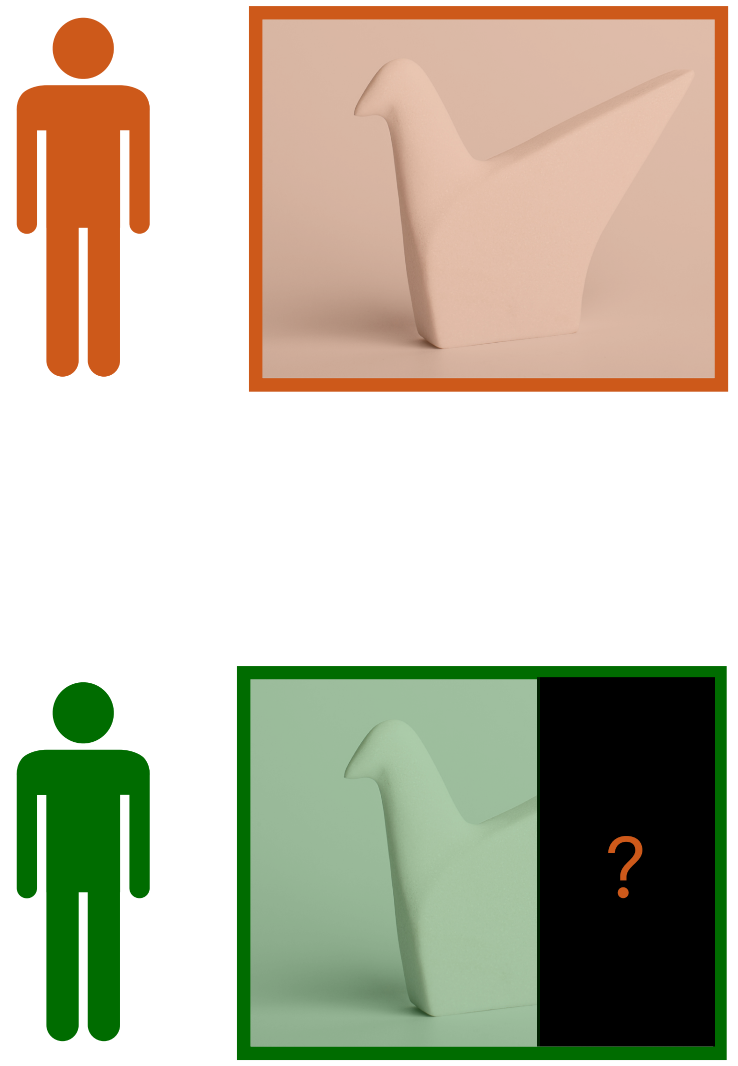


(**Informal**) **Theorem 1:** Let $\mathscr{M}$ be a posted price mechanism for the (private) independent values case with an approximation ratio of $\gamma > 1$. Consider an information structure with $\beta > 0$ and a fully informed seller ($\alpha = 1$). Then there **exists** a BIC **mechanism** $\mathscr{M}'$ with an **approximation** ratio of $\dfrac{2\gamma}{\beta}$.

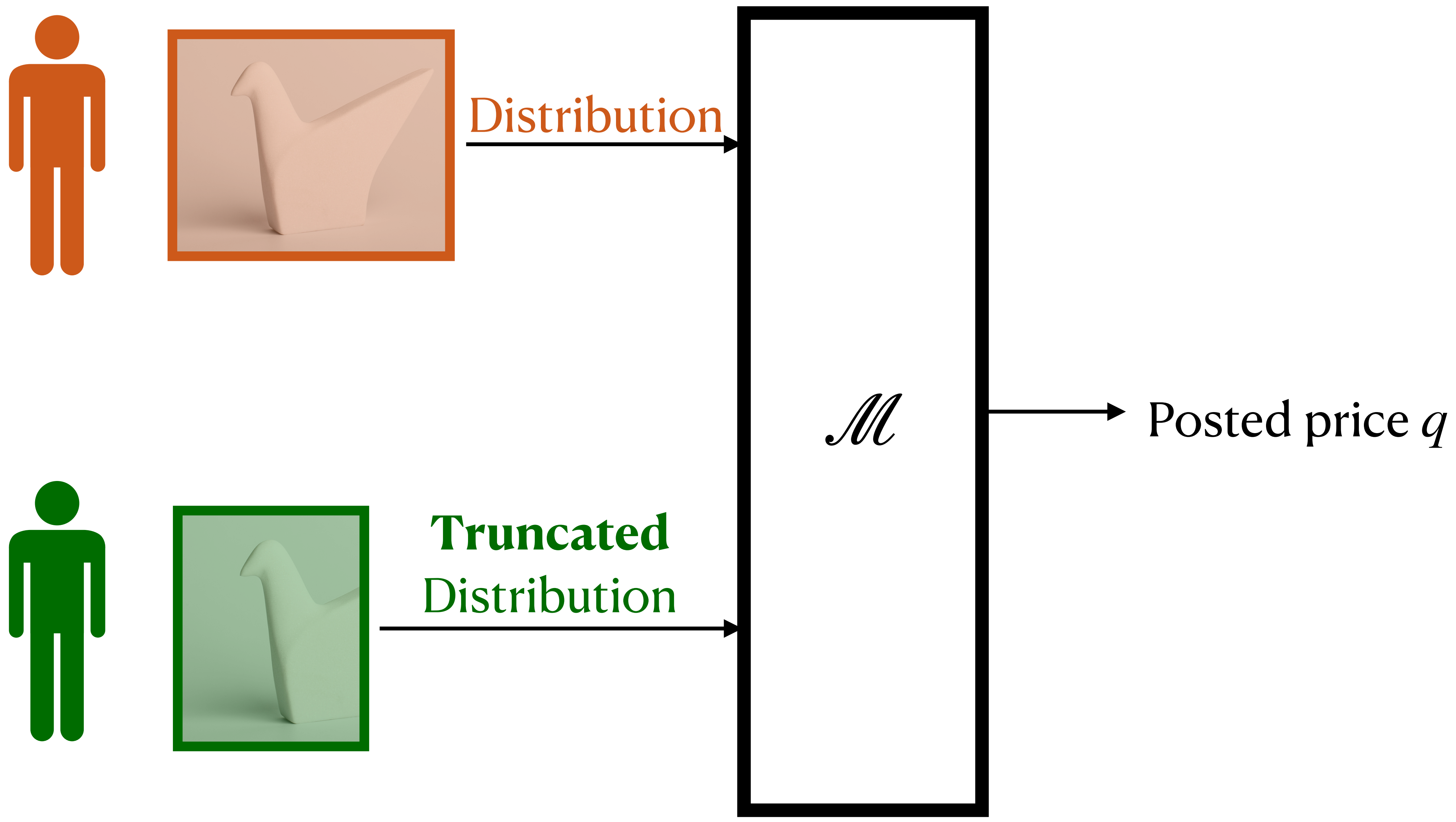
A Fully Informed Seller: (1, β) Edge



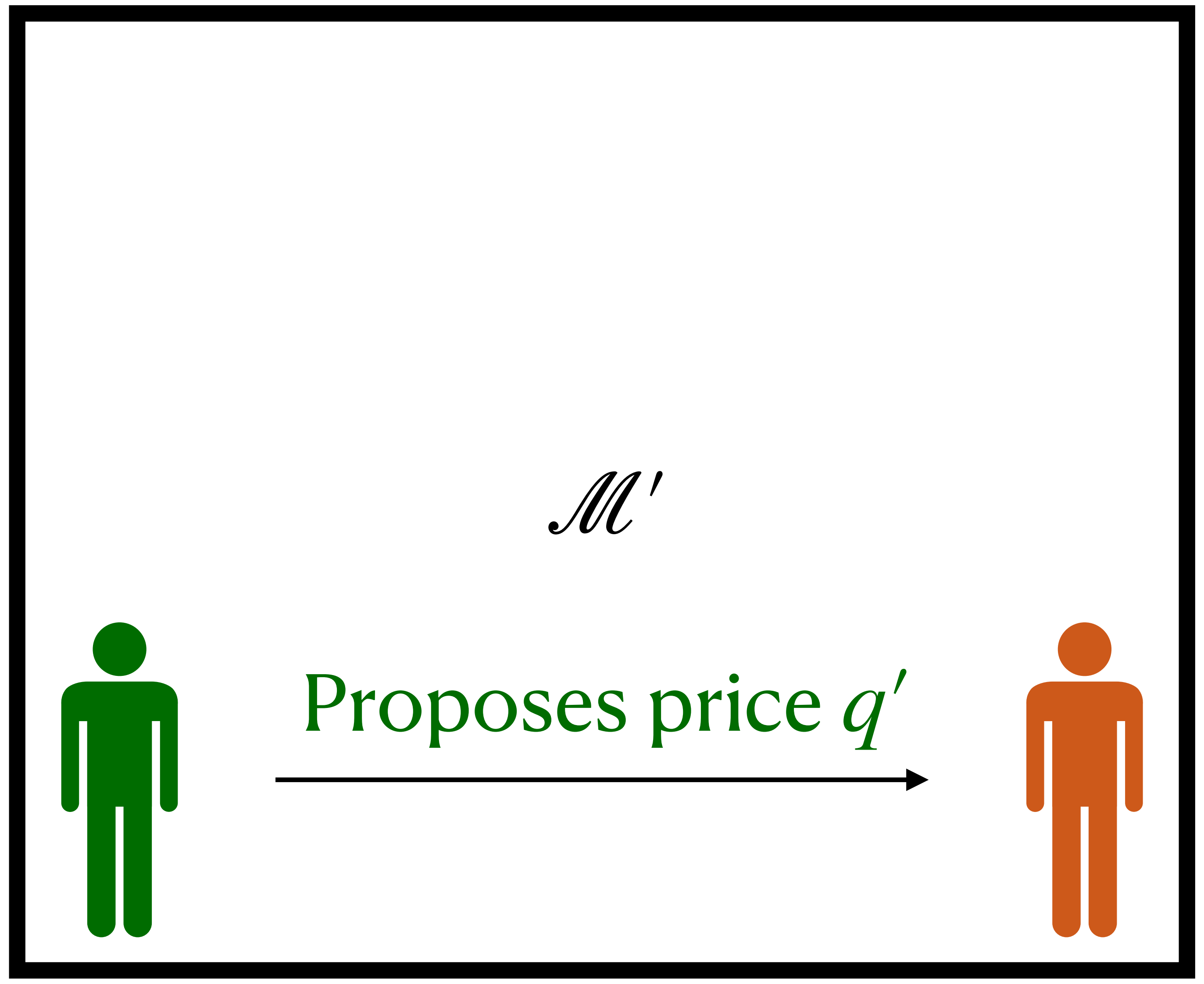
Proof Sketch - Compare Two Posted Price Mechanisms:



True Instance

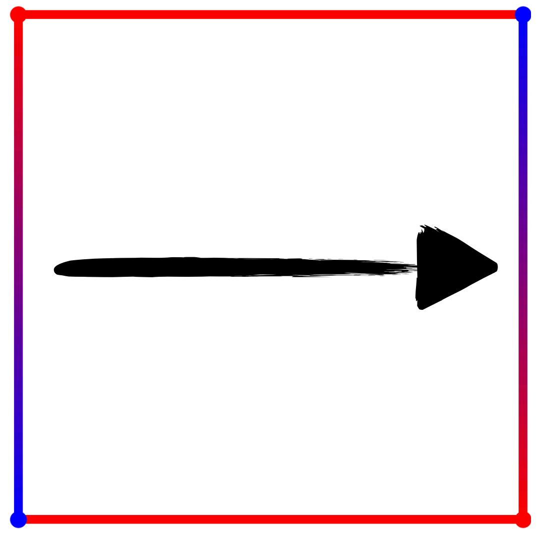


Independent Values Instance/Mechanism



Interdependent Mechanism

A Fully Informed Seller: (1, β) Edge



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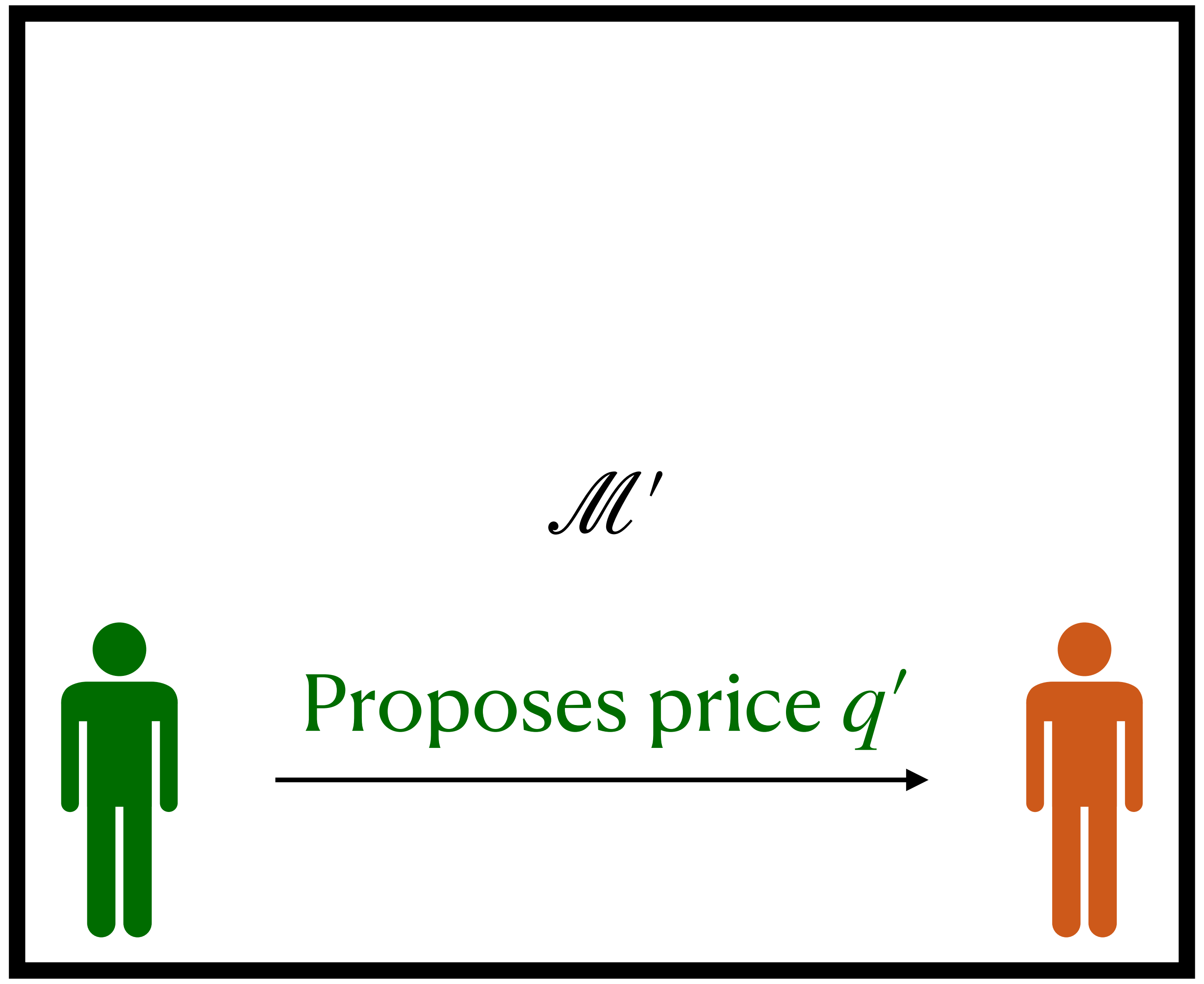
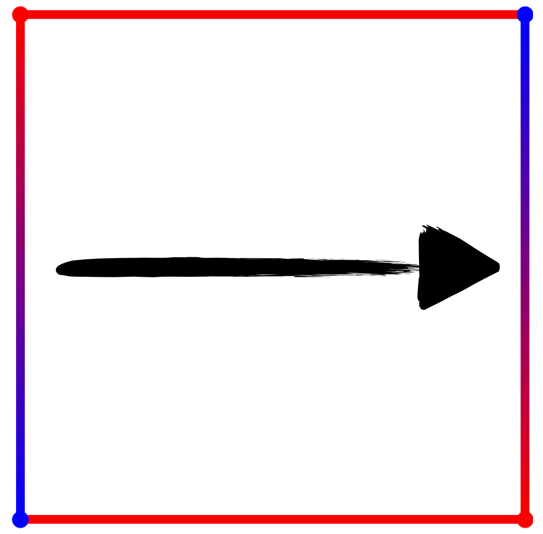


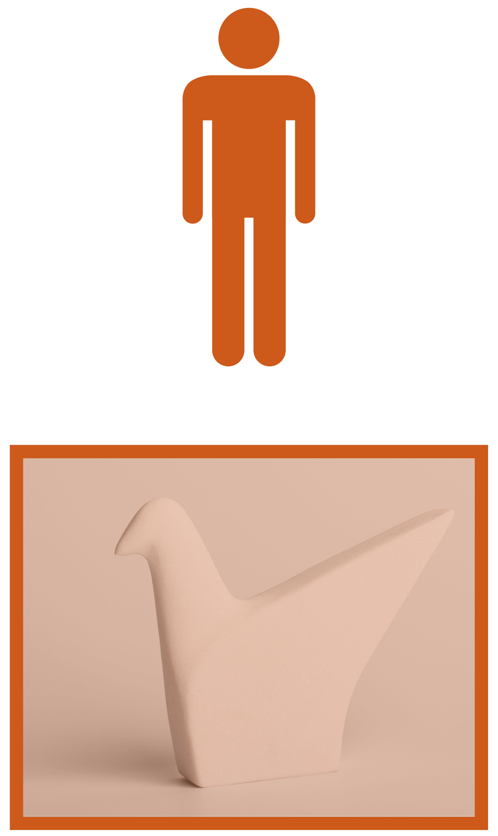
Investigating $\mathscr{M}'$:

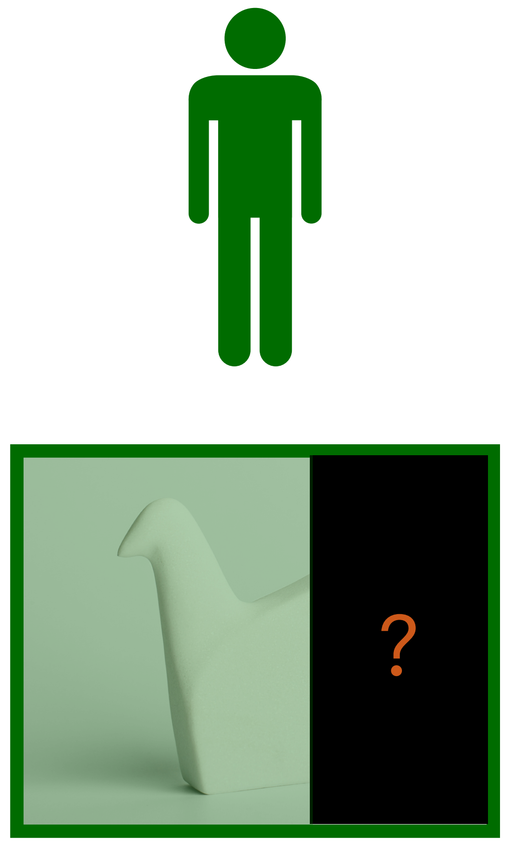
- The buyer proposes the price q' so $\mathscr{M}'$ satisfies **BIC** and **interim IR** (for the buyer).
- The seller is fully informed and responds to the proposed price optimally (so seller **BIC** and **interim IR** are also guaranteed).
- The **proposed price** q' can only be **higher** than price q (the price posted by the independent values mechanism $\mathscr{M}$).
- This implies the Welfare of $\mathscr{M}'$ is at **least as large** as the Welfare of $\mathscr{M}$.

A Fully Informed Seller: $(1, \beta)$ Edge



(Formal) Theorem 2: For every $\beta \in (0,1)$, there exists an information structure where the seller is fully informed and the buyer is β-informed, and **no** BIC and interim IR **mechanism can provide an approximation ratio** better than $\frac{2}{3\beta}$.



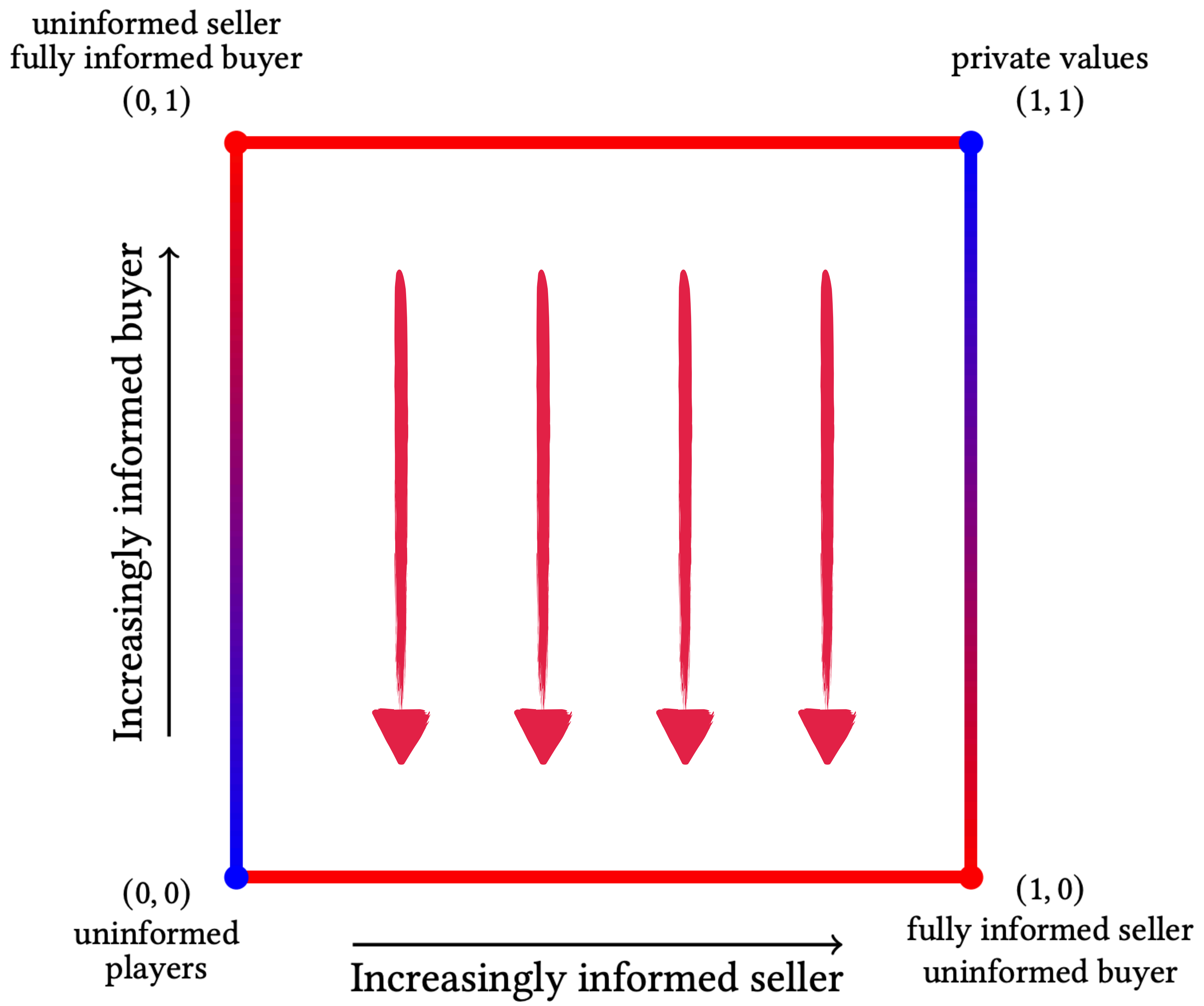


Positive result was:

$$\frac{2\gamma}{\beta}$$

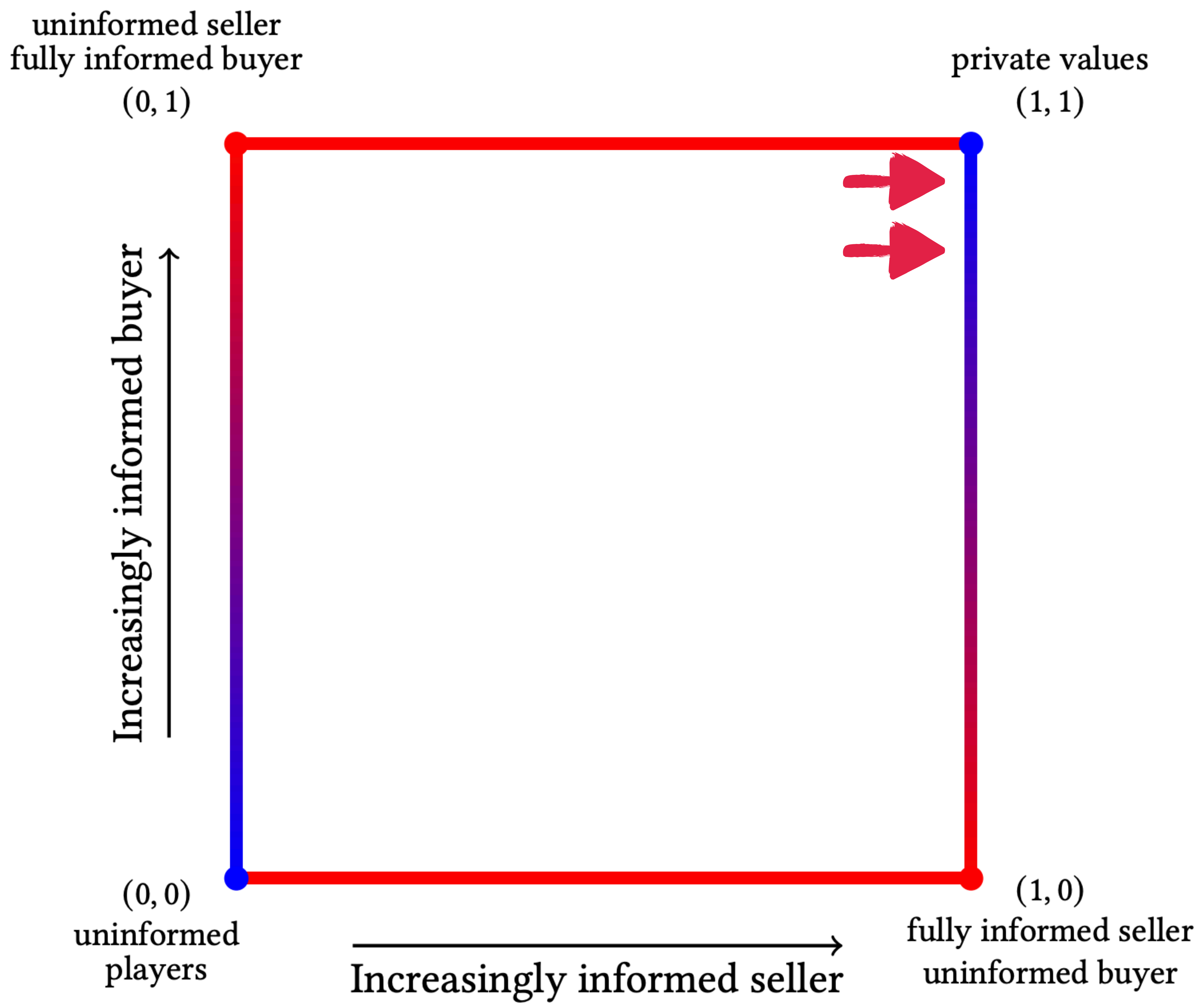
Interior of the square, overview of results

(Formal) Proposition 3: For every $\alpha > 0$ and $\beta < 1$, there exists an (α, β)-information structure where no BIC and interim IR mechanism can provide an approximation ratio better than $\frac{1}{2\beta}$.



Interior of the square, overview of results

(Formal) Proposition 4: For every $\alpha \in (0.9, 1)$ and $\beta \in [1-(1-\alpha)^3, 1)$, there exists an (α, β)-information structure where no BIC and interim IR mechanism can provide an approximation ratio better than $\frac{0.15}{1-\alpha}$.



Future Directions

1. **Tightly** characterize what is possible in the interior of the **square**.

2. Investigate what is possible for the **GFT** objective.

3. Study **specific families** of information structures.

4. Move beyond bilateral trade to **two-sided markets** (multiple buyers and/or sellers).

Thank you !