

Budget-Feasible Procurement: Optimizing for Welfare/Utility

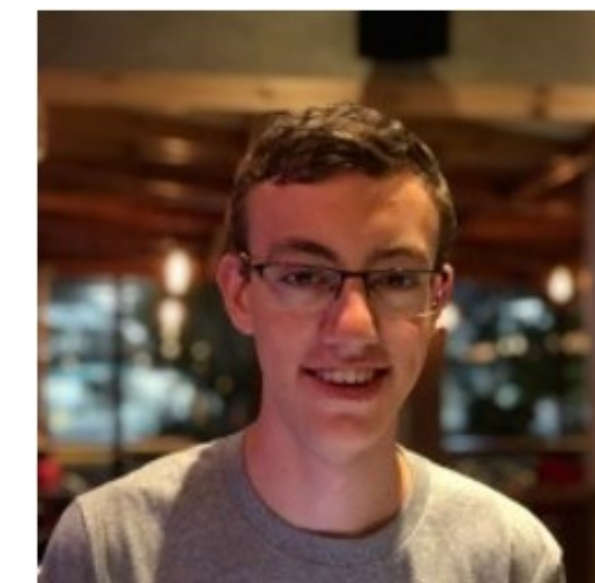
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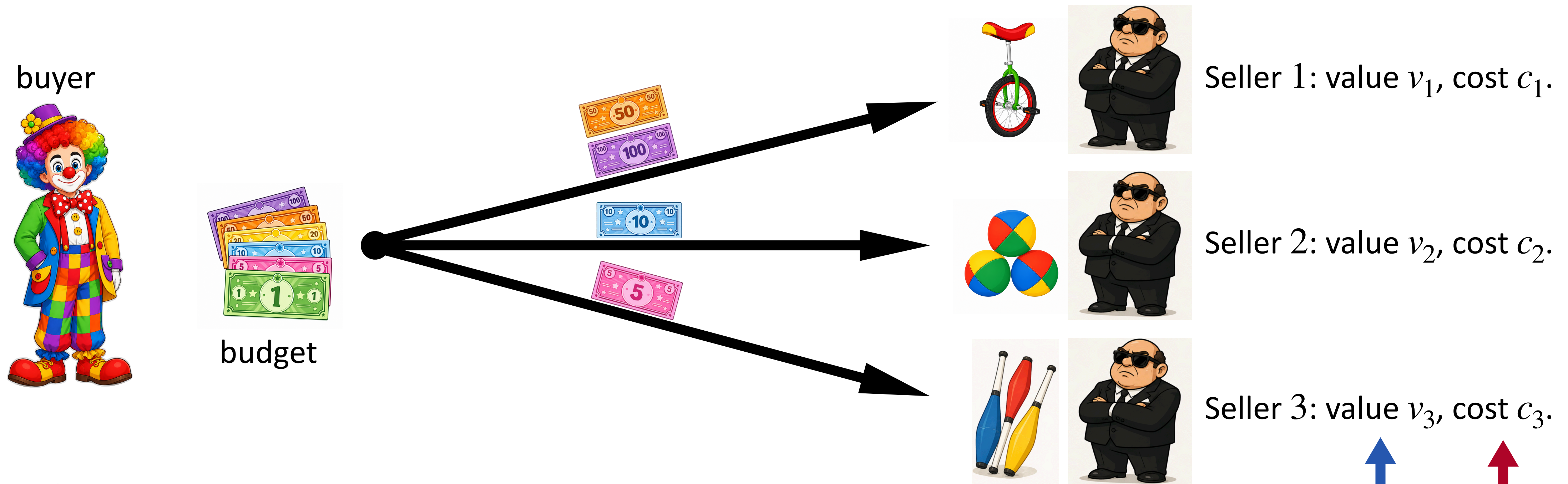


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Budget-Feasible Procurement



Setting:

- One **buyer** has budget B and faces n **sellers**.
- **Seller i** has (private) cost c_i and their good or service provides (public) **value v_i** .
- **Goal: Select items and pay** sellers while keeping total **payments $\leq B$** .
- Which **objective** should we be optimizing for?

Public Private

What if both were **public**?

Knapsack!

What objective to optimize for ?

- [Singer '10] introduced the problem as a **Value maximization** one (additive and combinatorial).
- Many **Value maximization** works since: [Chen Gravin Lu '11, BCGL12, AGN14, Ensthaler and Giebe '14, Balkanski Hartline '16, JM17, GJLZ20, AMSSTT25, BDLMP25, CFPT25] and more.

Our work [Eden Goldner Kerner T. '26]:

- **Question 1:** What if we take into account the sellers side as well?
 - ➡ Optimize for Social **Welfare** = Buyer Utility + Sellers Utility.
- **Question 2:** What if having a budget shouldn't mean spending all of it.
 - ➡ Optimize for Buyer **Utility**.



Optimizing Value



Optimizing Social Welfare



Optimizing Buyer Utility

Mechanism Design Task

A mechanism $\mathcal{M} = (x, p)$ elicits reported costs $\hat{\mathbf{c}}$ and must satisfy:

1. **DSIC-IR**,
2. **Budget feasibility** (never spend more than the budget B).

- Objectives:

1. **Value** of mechanism $\mathcal{M}$:

$$V(\mathbf{c}, \mathbf{v}, \mathcal{M}) = \sum_{i=1}^n x_i(\mathbf{c}) \cdot v_i .$$



2. **Welfare** of mechanism $\mathcal{M}$:

$$W(\mathbf{c}, \mathbf{v}, \mathcal{M}) = \sum_{i=1}^n x_i(\mathbf{c}) \cdot (v_i - c_i) .$$



3. **Utility** of mechanism $\mathcal{M}$:

$$U(\mathbf{c}, \mathbf{v}, \mathcal{M}) = \sum_{i=1}^n x_i(\mathbf{c}) \cdot v_i - p_i(\mathbf{c}) .$$



Prior-free welfare mechanism



Adapting the mechanism of [Chen Gravin Lu '11] for welfare $w_i = v_i - c_i$ instead of value v_i .

Mechanism sketch

1. **Filter sellers:** Keep i with $c_i \leq \min(v_i, B)$.
2. **High agent:** If $i^* = \operatorname{argmax}_i (v_i - c_i)$ is a $(1 + \sqrt{2})$ -approx to the optimal fractional solution, procure from i^* .
3. **Otherwise, many low:** greedily sort by welfare-per-cost $\frac{v_i - c_i}{c_i}$; procure from agents up to k , where $c_k \leq v_k \cdot \frac{B}{\sum_{j \leq k} v_j}$.
4. Charge **Myersonian** threshold payments.

✓ **DSIC-IR:** Monotone allocation, Myersonian payments, simple case analysis.

✓ **Budget-Feasibility:** Filtering step, threshold rule at step 3.

✓ **Approximation:** Knapsack style analysis, $(2 + \sqrt{2})$ -approx to the full information optimal.

What about Utility? No Prior-Free Guarantees

Instance: A single seller. $B = v = 1$. Consider a single posted price p ; the adversary picks cost c **after** seeing p .

Case 1

$p < 1$

- **Adversary** sets $c = (1 + p)/2$, so $p < c < 1$.
- Seller **refuses**; the **mechanism** gets utility 0.
- **Optimal** sets price of c , gets utility $1 - c$.

Case 2

$p = 1$

- **Adversary** sets $c = 0$.
- Seller **accepts**, the **mechanism** gets utility 0.
- **Optimal** sets price of ϵ , gets utility $1 - \epsilon$.

*In the prior-free setting, optimal utility is **inapproximable**!*



Bayesian Setting



Bayesian Utility: Soft- and Hard-Budget Problems

Bayesian: Costs c_i drawn independently from known distributions $\mathcal{F}_i$.

- **Soft-Budget problem:** Optimize expected utility, s.t. **expected** expenditure less than B .
- **Hard-Budget problem:** Optimize expected utility, s.t. expenditure **always** less than B .

Soft-Budget

$$\max_{x,p} \mathbb{E}_{\mathbf{c}} \left[\sum_{i \in [n]} (v_i x_i(\mathbf{c}) - p_i(\mathbf{c})) \right]$$

$$\text{s.t. } \mathbb{E}_{\mathbf{c}} \left[\sum_{i \in [n]} p_i(\mathbf{c}) \right] \leq B,$$

$(\mathbf{x}, \mathbf{p})$ is DSIC-IR.

Tractable Relaxation

Hard-Budget

$$\max_{x,p} \mathbb{E}_{\mathbf{c}} \left[\sum_{i \in [n]} (v_i x_i(\mathbf{c}) - p_i(\mathbf{c})) \right]$$

$$\text{s.t. } \sum_{i \in [n]} p_i(\mathbf{c}) \leq B, \quad \forall \mathbf{c}$$

$(\mathbf{x}, \mathbf{p})$ is DSIC-IR.

Want to solve this



Soft-Budget: The Tractable Relaxation

How to solve?

- Rewrite expected payments using the **virtual** cost functions $\phi_i(\mathbf{c})$. $\mathbb{E}_{\mathbf{c}}[p_i(\mathbf{c})] = \mathbb{E}_{\mathbf{c}}[\phi_i(c_i) \cdot x_i(\mathbf{c})]$
- Lagrangify the budget constraint. $\max_x \lambda B + \sum_i \mathbb{E}_{\mathbf{c}} \left[\left(v_i - (\lambda + 1) \phi_i(c_i) \right) x_i(\mathbf{c}) \right]$
- **Solution** for some λ : **Independent prices** $\tilde{p}_i$ for each agent i . $x_i^*(c) = \mathbb{1} \{ c_i \leq \phi_i^{-1}(v_i / (\lambda + 1)) \}$
- **Binary search** for λ^* ; **Optimal** solution to the Soft-Budget problem.

★ Idea from Bayesian **Value** maximization [Balkanski Hartline '16, Ensthaler Giebe '14].

★ **Important distinction**: In **Utility** the budget constraint is **not (necessarily) tight**.



Designing a Hard Budget Mechanism

Adapt the mechanism of [Balkanski Hartline '16] for **utility** instead of **value** .

Mechanism sketch

1. Solve for the **soft-budget** prices $\tilde{p}_i$.
2. Offer prices in **decreasing** order of **utility-per-price** $\frac{v_i - \tilde{p}_i}{\tilde{p}_i}$, until the budget runs out.

- By **tweaking** their **analysis**, we show that the **same guarantees** extend to **utility**.

- **DSIC-IR**, (hard)-**budget-feasible**, **approximation** ratio of $\left(1 - \frac{1}{k}\right) \left(1 - \frac{1}{\sqrt{2\pi k}}\right)$, where $k = \min_i \frac{B}{\tilde{p}_i}$.

- **Problem!** This is **not** a **constant** approximation (we could even have $k \leq 1$).



Designing a Hard Budget Mechanism (vol 2)

Mechanism sketch

1. **Filter Distributions:** Truncate cost distributions at B . Solve for the **soft-budget** prices $\tilde{p}_i$.
2. **Split agents:** High-price agents $H = \left\{ i : \tilde{p}_i \geq \frac{B}{\alpha} \right\}$ and low-price agents $L = \left\{ i : \tilde{p}_i < \frac{B}{\alpha} \right\}$.
3. Use the **best submechanism**:
 - I. Only H : Offer prices in **decreasing** order of **utility** $v_i - \tilde{p}_i$, until the budget runs out.
 - II. Only L : offer prices in **decreasing** order of **utility-per-price** $\frac{v_i - \tilde{p}_i}{\tilde{p}_i}$, until the budget runs out.

- The mechanism is also trivially **DSIC-IR** and (hard)-**budget feasible**.
- **Approximation ratio:** This mechanism is a 5-**approximation** to the optimal hard budget mechanism.



Paper Results TLDP(resent)



Welfare



1. A $(2 + \sqrt{2})$ -deterministic **prior free** mechanism.
2. A near matching lower **bound** of $(1 + \sqrt{2})$ for deterministic mechanisms.
3. **Generalization** for objectives $G \left(\sum x_i(\mathbf{c}) \cdot (v_i - \alpha \cdot c_i) \right)$, where $\alpha \geq 0$, G concave.



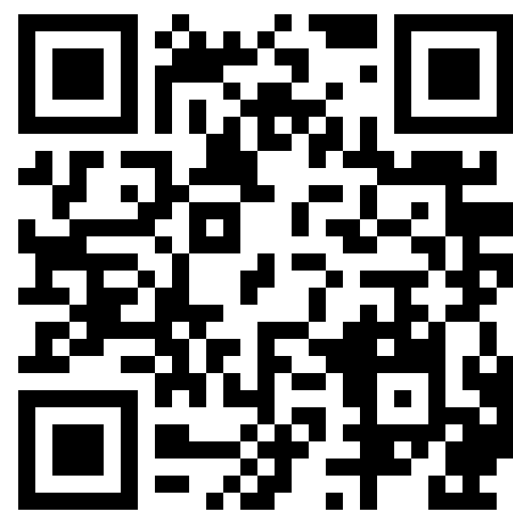
Utility



1. **Prior free impossibility** (single seller).
2. **Optimal** mechanism for **soft budget**, 5-**approximation** for **hard budget**.
3. **Tightness of analysis**, generalization for **irregular** distributions.
4. **Generalization** for objectives $\sum_{i=1}^n (a_i \cdot x_i(\mathbf{c}) \cdot v_i - b_i \cdot p_i(\mathbf{c}))$, where $a_i, b_i \geq 0$.

Takeaways

- **Value** budget feasible procurement **initiated** 16years ago and has been **studied thoroughly**.
- **Welfare** and **Utility** are equally **well motivated** objectives.
- **Old tricks** may still **work**, but **tweaks** and **turnarounds** seem **required**.
- In our paper, we provided **simple, deterministic, constant** factor approximations for the **additive** welfare and utility in budget feasible procurement and generalizations.



Questions?

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